2026

Four Year Under Graduate Programme (FYUGP)

4th Semester Examination (Under NEP)
(New Session 2024-25)

MATHEMATICS (Major)

Paper Code : MTMMJ DC-401

[Real Analysis-I]

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

[10 Marks]

1. Answer any *five* questions : 2×5=10

(a) Find the closure of the set

$$A=\left\{\frac{1}{n}:n=1, 2, 3, ...\right\}.$$

(b) Is the intersection of an infinite number of open sets in $\mathbb{R}$ open in $\mathbb{R}$? Justify your answer.

P.T.O.

(c) Is the sequence in $\mathbb{R}$

$$\sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}, \ldots$$

monotonically increasing? Justify your answer. 1+1

(d) If the series $\sum_{n=1}^{\infty} a_n$ converges, prove that $\lim_{n\to\infty} a_n = 0$.

(e) Is the function $f:\mathbb{R}\to\mathbb{R}$ defined by

$$f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right), & \text{if } x \neq 0, \\ 0, & \text{if } x = 0 \end{cases}$$

continuous at $x = 0$? Justify your answer.

(f) Show that the function $f:(0,1)\to\mathbb{R}$ defined by $f(x) = \cos\frac{1}{x}$ is not uniformly continuous on (0, 1).

(g) Prove that every differentiable function on $\mathbb{R}$ is continuous.

(h) Verify Rolle's theorem for $f(x) = x(1-x)$ in $[0,1]$.

Group - B

[40 Marks]

Answer any *four* questions : 10×4=40

2. (a) For every two real numbers a and b with $a < b$, prove that there exists a rational number r such that $a < r < b$.

   (b) Prove that a sequence in $\mathbb{R}$ is convergent if and only if it is a Cauchy sequence in $\mathbb{R}$. 4+6

3. (a) Prove that every bounded infinite set in $\mathbb{R}$ has at least one limit point in $\mathbb{R}$.

   (b) Test the convergence of the series

$$1+\frac{2^2}{3^2}+\frac{2^2 \cdot 4^2}{3^2 \cdot 5^2}+\frac{2^2 \cdot 4^2 \cdot 6^2}{3^2 \cdot 5^2 \cdot 7^2}+\ldots\ldots$$ 5+5

4. (a) Let f be a real-valued function continuous on $[a, b]$ and suppose that $f(a) \cdot f(b) < 0$. Show that there is at least one point $c \in (a,b)$ s.t. $f(c) = 0$.

   (b) A function $f:[0,1] \to \mathbb{R}$ is continuous on [0, 1] and assumes only rational values. If $f\left(\frac{1}{2}\right)=\frac{1}{2}$, prove that $f(x)=\frac{1}{2}\ \forall x \in [0,1]$. 6+4

5. (a) State and prove Cauchy's Mean Value Theorem.

P.T.O.

(b) Use the mean value theorem to prove

$$0<\frac{1}{x}\log\frac{e^x-1}{x}<1 \text{ for } x>0.$$

(c) Evaluate the limit : $\lim\limits_{x\to 0}\dfrac{e^x-e^{-x}-2\log(x+1)}{x\sin x}$.

5+3+2

6. (a) Let $f:\mathbb{R}\to\mathbb{R}$ be continuous at a point of $\mathbb{R}$, and let $f(x+y)=f(x)+f(y)$ for all $x,y\in\mathbb{R}$. Prove that f is uniformly continuous on $\mathbb{R}$.

(b) Does the limit $\lim\limits_{x\to 0}\sin\dfrac{1}{x}$ exist? Justify your answer.

(c) Use Taylor's theorem to prove

$$1+\frac{x}{2}-\frac{x^3}{8}<\sqrt{1+x}<1+\frac{x}{2} \text{ for } x>0.$$ 4+2+4

7. (a) Let $f:\mathbb{R}\to\mathbb{R}$ be defined by

$$f(x)=\begin{cases} x, & \text{if } x\in\mathbb{Q}, \\ 0, & \text{if } x\in\mathbb{R}\setminus\mathbb{Q}. \end{cases}$$

Show that f is continuous at the origin but discontinuous at other points in $\mathbb{R}$.

(b) A function f is differentiable on [0, 2] and $f(0)=0$, $f(1)=2$, $f(2)=1$. Prove that $f'(c)=0$ for some $c\in(0,2)$.

(c) Let $A=\left\{(-1)^m+\dfrac{1}{n}: m, n\in\mathbb{N}\right\}$. Show that 1 and -1 are the limit points of A.

5+3+2

2026

Four Year Under Graduate Programme (FYUGP)

4th Semester Examination (Under NEP)
(New Session 2024-25)

MATHEMATICS (Major)

Paper Code : MTMMJ DC-402

[Mechanics]

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Group - A

[10 Marks]

1. Answer any *five* questions : 2×5=10

(a) If $x = Ae^{t} + Be^{-t}$ is the displacement at time t of a particle, then find the acceleration.

(b) A particle moves according to the law

$$v = 4\,(x\sin x + \cos x)^{\frac{1}{2}}$$

where v is the velocity at a distance x. Find its acceleration.

P.T.O.

(c) A particle moves with a S. H. M., its position of rest being at a distance 'a' from the centre. Find by the principle of energy, the velocity at the centre.

(d) In a central force field show that $h = p.v$. The symbols have their usual meaning.

(e) If a particle describes the curve $r = a\cos\theta$ under a central force to the pole, prove that the force varies as $\frac{1}{r^5}$.

(f) For a simple harmonic motion $x = \cos\frac{\pi t}{3}$, find the periodic time.

(g) A particle describes a curve, whose polar equation is $r = 6e^{\theta}$. Show that radial and cross-radial velocity are same.

(h) State Kepler's laws of planetary motion.

Group - B

[40 Marks]

Answer any *four* questions : 10×4=40

2. (a) A particle moves under the action of central force $\mu u^2 + \lambda u^3$ per unit mass, where $u = \frac{1}{r}$. The velocity of projection at a distance R is V. Show

that the particle will ultimately go off to infinity if

$$V^2 > \frac{2\mu}{R} + \frac{\lambda}{R^2}.$$

(b) A particle is moving in a plane. Deduce the tangential and Normal acceleration of the particle.

5+5

3. (a) A particle is moving under a central force $F = F(u)$ and describe a nearly circular orbit. Find the condition of stability of the motion.

(b) A particle is projected at a distance 'a' with velocity $\sqrt{\mu / a}$ in a direction at right angles to the initial distance and subject to the action of a central force $\mu\left\{2(a^2+b^2)u^5 - 3a^2b^2u^7\right\}$. Show that the path is $r^2 = a^2\cos^2\theta + b^2\sin^2\theta$.

5+5

4. (a) A smooth table is capable of turning about one edge which is horizontal. Initially the table is horizontal and a particle rests on it at a distance 'a' from the fixed edge. If the table be now rotated downwards with an angular velocity ω, show that the particle will leave the table after a time 't' given by $a\sinh\omega t + \frac{g}{2\omega^2}\cosh\omega t = \frac{g}{\omega^2}\cos\omega t$.

(b) A particle is describing a circle of radius 'a' in such a way that its tangential acceleration is 'k' times

P.T.O.

the normal acceleration, where 'k' is a constant. If the speed of the particle at any point be u, prove that it will return to the same point after a time $\frac{a}{ku}\left(1-e^{-2\pi k}\right)$. 5+5

5. (a) Describe the motion of a particle with harmonic, damped and forced oscillations. 5

(b) Two bodies m_1 and m_2 are attached to the lower end of an elastic string whose upper end is fixed and are hung at rest; m_2 falls off. Show that the distance of m_1 from the upper end of the string at time t is $a+b+c\cos\left(\frac{g}{bt}\right)^{\frac{1}{2}}$, where a is the natural length of the string, b and $c\,(b>c)$ are the distances by which it would extended when supporting m_1 & m_2 respectively. 5+5

6. (a) Prove that for a particle of mass 'm' falling from rest under gravity from a height 'h' above the ground, the sum of the kinetic energy and the potential energy of the particle is constant at every point of its path.

(b) A particle is projected along the inner surface of a rough sphere and is acted on by no force. Show that it will return to the point of projection at the end of time $\frac{a}{\mu v}\left(ae^{2\mu\pi}-1\right)$, where a is the radius of the sphere, v is the velocity of projection and μ is the coefficient of friction. 5+5

7. (a) A body of mass $(m_1 + m_2)$ is split into two parts of masses m_1 & m_2 by an internal explosion which generate kinetic energy E; show that if after explosion the parts move in the same line as before, their relative speed is $\sqrt{\dfrac{2E(m_1+m_2)}{m_1 m_2}}$.

(b) A particle is projected vertically upwards, with velocity u in a medium whose resistance is μv^2 per unit mass, where v is the velocity of the particle and μ is a constant. Write down its equation of motion, and show that the particle comes to rest at a height $\dfrac{k^2}{2g}\left(1+\dfrac{\mu^2}{k^2}\right)$, above the point of projection where $k^2 = \dfrac{g}{\mu}$. What is the physical significance of constant k? 4+6

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2026

Four Year Under Graduate Programme (FYUGP)

4th Semester Examination (Under NEP)
(New Session 2024-25)

MATHEMATICS (Major)

Paper Code : MTMMJ DC-403

[Numerical Methods & C Programming Language]

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

[N.B. : Nonprogrammable Scientific Calculator may be used for numerical calculation]

Group - A

[10 Marks]

1. Answer any *five* questions : $2\times5=10$

(a) If $f(x)=4\cos x-6x$, find the relative percentage error in $f(x)$ for $x = 0$, if the error in $x = 0.005$.

(b) Prove that $\Delta\cdot\nabla=\Delta-\nabla$.

P.T.O.

(c) If for some $\alpha, \beta \in \mathbb{R}$, the integration formula $\int_0^2 f(x)dx = f(\alpha) + f(\beta)$ holds for all polynomials of degree at most 3, then find the value of $3(\alpha - \beta)^2$.

(d) Show that the initial approximation x_0 for finding $\frac{1}{N}$, where N is a positive integer, by the Newton-Raphson method, must satisfy $0 < x_0 < \frac{2}{N}$ for convergence.

(e) Show that the nth order divided difference of x^n is constant.

(f) What is the use of break and continue statement in a C program?

(g) Write the general form of 'if .... else' statement in C-language.

(h) Convert the mathematical expression $\sqrt[3]{x^2 + 7} + \sin^3 x$ into C expression.

Group - B

Answer any *four* questions : 10×4=40

2. (a) Show that the remainder in approximating $f(x)$ by the interpolation polynomial using distinct interpolating points $x_0, x_1, x_2, \ldots, x_n$ is of the form

$$(x - x_0)(x - x_1)(x - x_2)\ldots\ldots(x - x_n)\frac{f^{n+1}(\xi)}{(n+1)!}.$$

(b) Establish Newton's Backward Interpolation formula. (without error term). 5+5

3. (a) Obtain Composite Simpson's one-third rule for numerical integration.

(b) Prove that the expression for error term in Composite Simpson's one-third rule is given by

$$-\frac{b-a}{180}h^4 y^{(iv)}(\xi),\ a<\xi<b.$$ 5+5

4. (a) (i) Derive the condition of convergence of Newton-Raphson method in finding a real root of $f(x)=0$.

(ii) Give the geometrical interpretation of Newton-Raphson method.

(b) Compute $y(0.3)$ from the equation $\frac{dy}{dx}=x-y,\ y(0)=1$ taking $h = 0.1$ by fourth-order Runge-Kutta method. (3+2)+5

5. (a) Write a C-program to find the sum of the series $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\ldots\ldots\infty$ satisfying the tolerance level : $|\text{Term}| < 0.0000001$.

(b) Evaluate $\int_{-1}^{3}|x|\,dx$ numerically by Trapezoidal and Simpson's $\frac{1}{3}$rd method taking four equal subintervals. Which method gives better results and why? 5+(4+1)

P.T.O.

6. (a) Write a C-program to find the mean and variance of the following frequency distribution :

$$x : x_1 \; x_2 \; x_3 \ldots\ldots x_n$$
$$f : f_1 \; f_2 \; f_3 \ldots\ldots f_n$$

(b) Describe Gauss-Jacobi method for numerical solution of a system of linear equations $Ax = b$, where $A = \left[a_{ij}\right]_{n \times n}$, $x = \left(x_1, x_2, \ldots, x_n\right)^T$, $b = (b_1, b_2, \ldots\ldots, b_n)^T$.

Prove that in matrix form, the method can be written as $x^{(k+1)} = -D^{-1}\left(L+U\right)x^{(k)} + D^{-1}b$, where the notations have their usual meaning. 5+5

7. (a) Find the greatest eigen value of the matrix

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

correct to four significant figure, and corresponding eigen vector using power method.

(b) Write a C-program to find the transpose of a matrix. 6+4
